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Since $f(a+x) \equiv (a+x)^{n+1} - a^{n+1} \equiv \binom{n+1}{1}a^n x + \binom{n+1}{2}a^{n-1}x^2 + \cdots + \binom{n+1}{i+1}a^{n-i}x^{i+1} + \cdots + x^{n+1}$, and again,

$$f(a+x) \equiv f(a) + xf'(a) + \frac{x^2}{\underline{|2}}f''(a) + \cdots + \frac{x^{i+1}}{(i+1)!}f^{(i+1)}(a) + \cdots + \frac{x^{n+1}}{(n+1)!}(n+1)!,$$

we have, on comparison of coefficients,

$$\frac{1}{(i+1)!}f^{(i+1)}(a) = \binom{n+1}{i+1}a^{n-1} = \frac{(n+1)!}{(i+1)!(n-i)!}a^{n-1},$$

$$f^{(i+1)}(a) = (i+1)\phi^{(i)}(a) = (i+1)S_{i,\,n}\cdot a^{n-1} = \frac{(n+1)!}{(n-i)!}a^{n-1},$$

so that

$$S_{i,\,n} = \frac{1}{i+1}\cdot\frac{(n+1)!}{(n-i)!},$$

and $(n+1)!/(n-i)!$ is the last term of $S_{i+1,\,n+1} = \sum_{k=1}^{k=n-i+1}(i+k)!/(k-1)!$.

Also solved by HORACE OLSON.

457. Proposed by FRANK IRWIN, University of California.

If a be any number prime to m and m/a be developed as a continued fraction,

$$\frac{m}{a} = a_1 + \frac{1}{a_2+}\frac{1}{a_3+}\cdots+\frac{1}{a_{k-1}+}\frac{1}{a_k},$$

with $a_1 \neq 0$, then there will exist a number b such that

$$\frac{m}{b} = a_k + \frac{1}{a_{k-1}+}\cdots+\frac{1}{a_2+}\frac{1}{a_1}.$$

Show that $ab \equiv \pm 1 \pmod{m}$ and determine the sign.

SOLUTION BY THE PROPOSER.

Write m and a as functions of the a's: $m = [a_1, a_2, \cdots a_k]$, $a = [a_2, a_3, \cdots a_k]$, using the *Gaussische Klammer* notation, see BACHMANN, *Niedere Zahlentheorie*, vol. 1, p. 104; or, with a different symbol, $K(a_1, a_2, \cdots a_k)$, CHRYSTAL's *Algebra*, part II, 2d ed., p. 495.

Similarly, $b = [a_{k-1}, \cdots a_2, a_1]$, and if p_{k-1}/q_{k-1} be the next to last convergent to m/a, $p_{k-1} = [a_1, a_2, \cdots a_{k-1}]$. But, by an elementary property of these expressions, $[a_1, a_2, \cdots a_{k-1}] = [a_{k-1}, \cdots a_2, a_1)$; that is, $p_{k-1} = b$.

Again, since p_{k-1}/q_{k-1}, and m/a are successive convergents, $mq_{k-1} - ap_{k-1} = (-1)^k$; that is, $mq_{k-1} - ab = (-1)^k$, or $ab \equiv (-1)^{k-1} \pmod{m}$.

GEOMETRY.

483. Proposed by LAENAS G. WELD, Pullman, Illinois.

A circle is inscribed in a triangle. In each of the three spandrils exterior to the circle another circle is inscribed; in the remaining spandrils three other circles; and so on ad infinitum. Show that the sum of the areas of these circles is given by the formula:

$$\Sigma = \frac{\pi}{4}\cdot\frac{\Delta^2}{S^2}\left[\frac{1}{\sin(A/2)} + \frac{1}{\sin(B/2)} + \frac{1}{\sin(C/2)} - 2 + \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2}\right].$$

SOLUTION BY J. A. CAPARO, University of Notre Dame.

Let A, B, C and a, b, c denote the angles and sides opposite these angles of the given triangle. Since the center of the inscribed circle is at the intersection of the bisectors of the angles of the triangle, the centers of the circles inscribed in the spandrils are on these bisectors. Let R be the radius of the inscribed circle and R_1, R_2, $\cdots$ R_n the radius of the circles whose centers are A_1, A_2, $\cdots$ A_n, respectively.